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THE WELFARE IMPLICATIONS OF COSTLY MONITORING IN THE CREDIT MARKET: A NOTE*

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Hillier and Worrall (1994) derived a surprising result that credit should be further rationed in the costly-monitoring credit-rationing equilibrium. This note shows that their result may be reversed if monitoring costs are endogenously determined.

In a paper published in this JOURNAL, Hillier and Worrall (1994, henceforth HW) derived a result that the establishment of market efficiency in the credit-rationing equilibrium of Williamson (1987) requires a policy aiming at tightening credit rationing. Because this result is surprising and implies unconventional policy prescriptions, it is worthwhile to check its robustness in a more general setting. In this note, we extend the HW model to allow for endogenous determination of monitoring costs and show that the HW result can be reversed. The general equilibrium model developed in this note provides a better foundation for policy analysis in the costly monitoring situation.

1. The Model

This section extends the HW model to allow for endogenous determination of monitoring costs. Consider an economy with α capitalists and $(1-\alpha)$ entrepreneurs. Capitalists can either invest K_y in a 'safe' sector to obtain output $Y = Y(K_y)$, $Y' > 0$, $Y'' < 0$, or lend (through competitive banks) capital to entrepreneurs who can undertake projects in a 'risky' sector. Each project requires one entrepreneur and k units of capital (k is an exogenous number), and the output per unit of capital is a random variable x that follows a distribution function $F(.)$ over $[0, x_{max}]$. Let $\overline{x}$ be the mean of x. Assuming i.i.d. production risk and a large number of entrepreneurs, $\overline{x}$ is a constant. Denote K_x and X as the capital and output of the risky sector, $X(K_x) = \overline{x}K_x$.

Capitalists do not observe x unless they monitor. In the HW model, monitoring cost per unit of loan is assumed to be a fixed amount of effort. To endogenise the determination of monitoring costs, we introduce into the model a sector that produces monitoring service whose production function is given by $Z = Z(K_z)$, $Z' \geqslant 0$, $Z'' \leqslant 0$. Letting c be the market price of Z and assuming one unit of Z is required to monitor one unit of loan, monitoring cost per unit of loan equals c. The endogenous determination of c as the price of monitoring service distinguishes our model from that of Hillier and Worrall (1994).

The existing literature (Townsend, 1979; Gale and Hellwig, 1985; William-

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son 1987) has established the result that the optimal contract in the presence of costly monitoring is a debt contract where monitoring occurs only in the event of default. Specifically, for an entrepreneur who borrows capital, if she claims that her x is below R, the bank monitors and confiscates the entire outcome; otherwise no monitoring occurs and she pays R per unit of loan. Thus total demand for monitoring service equals $K_x F(R)$. The market for monitoring service clears when $K_x F(R) = Z(K_z)$.

Loan payment R is chosen by banks to maximise the expected profit per unit of loan, $\pi = \xi(R) - cF(R) - r$, where $\xi(R) = R[1 - F(R)] + \int_0^R x dF(x)$ is the expected revenue, $cF(R)$ is the expected monitoring costs, and r is the deposit rate. Banks take c and r as given, and solve a profit-maximising R_m from the first-order condition $\xi'(R_m) = cF'(R_m)$. Competition drives banks' profit to zero in equilibrium; hence $r_m = \xi(R_m) - cF(R_m)$.

Given r_m, investment in the safe sector is determined by $Y'(K_y) = r_m$, and investment in monitoring service is determined by $cZ'(K_z) = r_m$. The supply of loanable funds is then equal to $K_x^s = K - K_y - K_z$, where K denotes total capital in the economy. *Ex ante*, all entrepreneurs want to borrow from banks, provided that $\overline{x} > \xi(R_m)$, which is assumed to hold. The demand for loans is then given by $K_x^d = (1 - \alpha) k$. A credit-rationing equilibrium emerges when the profit-maximising r_m is not high enough to attract loanable funds for all entrepreneurs.

2. Welfare Implications

This section compares the equilibrium allocation of credit to entrepreneurs (K_x^m) with the socially-efficient allocation (K_x^*). The socially-efficient allocation requires that marginal social benefit be equated to marginal social cost,

$$\overline{x} = r^* + c^* F(R^*) + c^* K_x^* \frac{dF(R^*)}{dK_x^*}, \tag{1}$$

where an asterisk denotes the efficient level of the variable. The left hand side show the marginal social benefit from investment in the risky sector. The right hand side shows the marginal social cost, which is the sum of the marginal capital cost as measured by r^* and the *marginal* monitoring costs as measured by the last two terms in the equation. In contrast, the equilibrium allocation only considers the *average* monitoring cost, which is clear from the zero-profit condition of banks, $\xi(R_m) = r_m + cF(R_m)$. Thus, the socially-efficient allocation takes into account the externality of a change in K_x on the default probability $F(R)$, but the equilibrium allocation does not.

What is the implication of the externality on credit allocation? Under the assumption of exogenous monitoring costs (c), Hillier and Worrall (1994) showed that $K_x^m > K_x^*$, i.e., *too much* credit is allocated to entrepreneurs in the market equilibrium even though they are rationed in credit. With endogenously-determined monitoring costs, however, the HW result may not hold. An example suffices to show the possible reversal of the HW result.

Suppose x follows the uniform distribution over [0, 1] so that $F(R) = R$, $\overline{x} = 0.5$, and $\xi(R) = R - 0.5R^2$. Let $Y(K_y) = \ln(K_y)$ and $Z(K_z) = K_z$. The market equilibrium can be solved from: (1) Banks' zero-profit condition, $r = R - 0.5R^2 - cR$; (2) Profit maximisation in the safe sector, $1/K_y = r$; (3) Profit maximisation in monitoring service, $c = r$; (4) Equilibrium in monitoring service, $K_x R = K_z$; (5) Resource constraint, $K_x + K_y + K_z = K$; (6) Banks' profit maximisation, $1 - R - c = 0$. The solution is shown in the first column of Table 1.

Table 1
Solutions to the Example

Equilibrium allocation	Efficient allocation
$K_x^m = (K - 2 - \sqrt{3})/\sqrt{3}$	$K_x^* = [K - (2 + \sqrt{3 + s^2})/(1 - s^2)]/(\sqrt{3 + s^2} - s)$
$K_y^m = 2 + \sqrt{3}$	$K_y^* = (2 + \sqrt{3 + s^2})/(1 - s^2)$
$K_z^m = (K - 2 - \sqrt{3})(1 - 1/\sqrt{3})$	$K_z^* = [K - (2 + \sqrt{3 + s^2})/(1 - s^2)][1 - 1/(\sqrt{3 + s^2} - s)]$
$R_m = \sqrt{3} - 1$	$R^* = \sqrt{3 + s^2} - 1 - s$
$r_m = 2 - \sqrt{3}$	$r^* = 2 - \sqrt{3 + s^2}$
$c_m = 2 - \sqrt{3}$	$c^* = 2 - \sqrt{3 + s^2}$

The socially-efficient allocation can be solved from (1)–(5) plus: (7) Market efficiency condition, $r + cR + cK_x dR/dK_x = \overline{x} = 0.5$. By differentiating (1)–(5) we obtain $dR/dK_x = (1 + R)/\{(1 - R - c)/[r^2(1 + R)] - K_x\}$. We write equation (7) as $1 - R - r = s$, where $s \equiv 0.5 - (1 - c)R + (1 + R)cK_x/(1 - (R - c)/[r^2(1 + R)] - K_x)$. Treating s as a parameter, we obtain the solution shown in the second column of Table 1. Substituting the solution into the expression for s, we obtain K as a function of s. To satisfy the restrictions $R \in [0, 1]$ and $K \geqslant K_y^*$, s must be in the range of [0, 1]. It can be verified that K is positive ($K \geqslant 2 + \sqrt{3}$) and is monotonically increasing in s over $s \in [0, 1]$.

The difference between K_x^* and K_x^m is given by

$$DK \equiv K_x^* - K_x^m = \frac{K - (2 + \sqrt{3 + s^2})/(1 - s^2)}{\sqrt{3 + s^2} - s} - \frac{(K - 2 - \sqrt{3})}{\sqrt{3}}. \qquad (2)$$

Using $K = K(s)$, we obtain DK as a function of s. Fig. 1 depicts the function. It is found that $DK \leqslant 0$ for $s \in [0, 0.39]$, and $DK > 0$ for $s \in [0.39, 1]$. The critical value $s = 0.39$ corresponds to $K = 7.28$. Thus, if $2 + \sqrt{3} < K < 7.28$, then $DK < 0$, and the HW result holds. If $K > 7.28$, then $DK > 0$, and the HW result is reversed. Two numerical examples in Table 2 ($s = 0.3$ in case 1 and $s = 0.5$ in case 2) illustrate the two possibilities.

In this particular example, the HW result is reversed when the economy has sufficiently large K. The intuition is as follows. As the social planner reduces R to internalise the externality, both c and K_z fall and capital releases from the monitoring sector. The social planner chooses the efficient allocation of capital to the safe sector based on r^*. In this example, $r^* < r^m$ so that $K_y^* >$

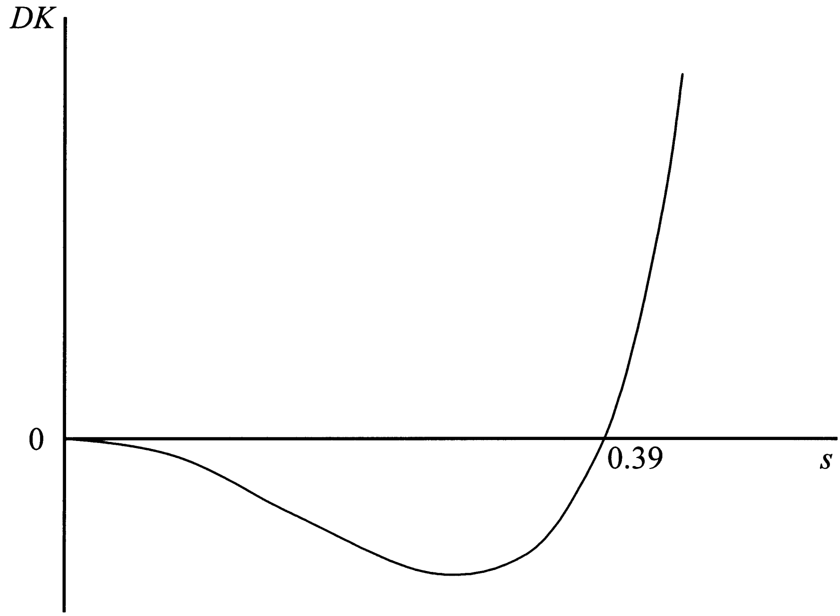

Fig. 1.

Table 2
Two Possible Cases

Case 1. $K = 5.63$	K_x	K_y	K_z	R	r	c
Equilibrium allocation	1.10	3.73	0.80	0.73	0.27	0.27
Efficient allocation	1.03	4.13	0.47	0.46	0.24	0.24
Case 2. $K = 11.32$						
Equilibrium allocation	4.38	3.73	3.21	0.73	0.27	0.27
Efficient allocation	4.80	5.07	1.45	0.30	0.20	0.20

K_y^m.[1] If K is large, then the capital released from the monitoring sector exceeds $(K_y^* - K_y^m)$ and therefore $K_x^* > K_x^m$. If K is small, however, the increase from K_y^m to K_y^* exceeds the capital released from the monitoring sector and therefore $K_x^* < K_x^m$.[2] By contrast, in the HW model with exogenous c and zero capital investment in monitoring service, the economy is characterised by $r^* < r^m$ and $K_z^* = K_z^m = 0$ such that the efficient allocation is $K_y^* > K_y^m$ and $K_x^* < K_x^m$.

[1] When $Z(K_z)$ takes a more general functional form, it is possible that $r^* > r_m$ and therefore $K_y^* < K_y^m$. In that case, both K_y^* and K_z^* are lower than their equilibrium levels and the reversal of the HW result is independent of K (but dependent of parameters of production functions).

[2] The equilibrium solution is efficient only when $s = 0$, which corresponds to $K = 2 + \sqrt{3}$. In that case, all capital is invested in the safe sector.

3. Conclusion

Does credit need to be further rationed in the costly-monitoring credit-rationing equilibrium? The answer by Hillier and Worrall (1994) is affirmative. By extending the HW model to allow for endogenous determination of monitoring costs, we show that the HW result may be reversed. The HW model has an externality that leads to excessive monitoring, but not necessarily excessive credit to entrepreneurs.

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