Journal of International Economics 54 (2001) 5–25

Journal of INTERNATIONAL ECONOMICS

www.elsevier.nl/locate/econbase

Factor bias, sector bias, and the effects of technical progress on relative factor prices

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Received 25 January 1998; received in revised form 30 December 1999; accepted 28 February 2000

Abstract

The effects of technical progress on relative factor prices in the two-country, two-good, two-factor Heckscher-Ohlin model are investigated. Technical progress is classified according to factor-augmenting bias, factor-using bias, and sector bias. A complete set of relations between technical progress parameters and relative factor prices, depending on the elasticities of substitution in demand and in production, are worked out. The relative importance of factor bias and sector bias of technical change in determining relative factor prices are examined. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Heckscher-Ohlin; Factor prices; Technical change; Factor bias; Sector bias

JEL classification: F11

1. Introduction

In recent studies on rising wage inequality in advanced countries, there has been some dispute regarding the relevance of factor bias (e.g. skill-using or labor-using) and sector bias (e.g. skill-intensive sector or labor-intensive sector) of technical progress in determining relative factor prices. Leamer (1998) and Krugman (2000) have expressed two opposite views, with Leamer arguing that sector bias is all that matters for relative wages and Krugman showing a case in which factor bias is all

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PII: S0022-1996(00)00089-1

that matters. Not surprisingly, the two benchmark cases were derived from special assumptions. Leamer assumed technical progress that is local and that occurs in a small open economy, while Krugman assumed technical progress that occurs in an integrated world economy with Cobb-Douglas preferences and Leontief technologies.[1]

Motivated by the debate, this paper aims to derive results on a wider range of specifications of preferences, technologies, and technical progress. We consider: (1) homothetic preferences with different elasticities of substitution in demand, (2) linear homogeneous technologies with different elasticities of factor substitution, (3) factor-augmenting bias and factor-using bias, (4) local technical progress, identical global technical progress, and non-identical global technical progress.

We conduct our analysis in the two-by-two Heckscher-Ohlin model. Technical progress in this model has been studied in some classic papers, notably Findlay and Grubert (1959) and Jones (1965). Findlay and Grubert (1959) derived the impacts of technical progress on output levels and commodity prices, while Jones (1965) showed the relations between technical progress and factor prices. The recent studies on technical progress and relative wages are applications and extensions of these earlier analyses.

This paper complements the existing literature in two respects. First, we define factor-augmenting bias as well as factor-using bias. While Hicks' (1932) definition of factor-using bias is theoretically appealing, factor-augmenting parameters are explicit in production and cost functions and are often useful in empirical and computational studies of technical progress.[2] In this paper we work out a set of relations between factor-augmenting parameters and relative factor prices (Table 1). We also link factor-augmenting parameters to factor-using technical progress and obtain a set of relations between factor-using technical progress and relative factor prices (Table 2).

Second, we examine global technical progress that is non-identical across countries. In the integrated world economy model (e.g. Krugman, 2000), global technical progress is identical across countries. Davis (1998) pointed out that empirical studies have confirmed the existence and importance of cross-country technical differences at the industry level (e.g. Jorgenson, 1995). He considered both global and local technical progress in the two-by-two model and derived results for the case of Leontief production technology. Our analysis extends Davis' results to non-Leontief production technologies. Moreover, we follow the spirit of Davis (1998) to draw a distinction between identical and non-identical global technical progress. This distinction helps to clarify the issue of factor bias vs.

[1]The assumption of Leontief technologies is not essential for Krugman's (2000) result.

[2]Indeed, factor-augmenting parameters are useful when estimating the effects of factor-using technical progress (Binswanger, 1974). One advantage of using factor-augmenting parameters to measure technology bias is that it maintains the properties of linear homogeneity of the neoclassical production function (Sato, 1970).

Table 1
The effects of factor-augmenting technical progress[a]

Factor bias	Sector bias	Preferences	Small open economy	Integrated world economy			Large open economy		
				$\sigma_i < 1$	$\sigma_i = 1$	$\sigma_i > 1$	$\sigma_i < 1$	$\sigma_i = 1$	$\sigma_i > 1$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Labor-Augmenting	X	$\eta = 1$	−	−	0	+	−	−	+/−
		$\eta > 1$	−	−	−	+/−	−	−	+/−
		$\eta < 1$	−	+/−	+	+	+/−	+/−	+/−
Skill-Augmenting	X	$\eta = 1$	−	+	0	−	+/−	−	−
		$\eta > 1$	−	+/−	−	−	+/−	−	−
		$\eta < 1$	−	+	+	+/−	+/−	+/−	+/−
Labor-Augmenting	Y	$\eta = 1$	+	−	0	+	+/−	+	+
		$\eta > 1$	+	+/−	+	+	+/−	+	+
		$\eta < 1$	+	−	−	+/−	+/−	+/−	+/−
Skill-Augmenting	Y	$\eta = 1$	+	+	0	−	+	+	+/−
		$\eta > 1$	+	+	+	+/−	+	+	+/−
		$\eta < 1$	+	+/−	−	−	+/−	+/−	+/−

[a] Notes: '+', '0', '−', and '+/−' refer to positive, zero, negative, and indefinite effect of the corresponding technical progress on the relative wage of unskilled labor. X and Y refer to the skill-intensive sector and the labor-intensive sector, respectively. η is the elasticity of substitution in demand. σ_i is the elasticity of factor substitution in sector i where the technical progress occurs, $i = X$, Y.

sector bias. In particular, we find that the key condition for Krugman's (2000) case (in which only factor bias matters for relative wages) is that technical progress is not just global, but global *and* identical across countries. When technical progress occurs at different rates in the two countries, the sector bias also matters for relative factor prices even under Cobb-Douglas preferences.

We organize the remainder of the paper as follows. In Section 2 we specify the two-by-two Heckscher-Ohlin model, using factor-augmenting parameters to characterize production technologies. In Section 3, we derive a set of relations between technical progress parameters and relative factor prices and examine the roles of factor bias and sector bias. Section 4 concludes.

2. The model

In this section we present the $2\times2\times2$ Heckscher-Ohlin model, allowing for cross-country technical differences. We label the two countries Home and Foreign, the two goods X and Y, and the two factors skilled labor and unskilled labor. Home (Foreign) has fixed endowments of H (H^*) units of skilled labor and L (L^*) units of unskilled labor. Factors are perfectly mobile within each country but are immobile between countries. We assume that good X is more intensive in skilled labor than good Y at all relevant ratios of skilled to unskilled wage. Our analysis

Table 2
The effects of factor-using technical progress[a]

Factor bias (1)	Sector bias (2)	Preferences (3)	Small open economy (4)	Integrated world economy (5)	Large open economy (6)
Skill-Using	X	$\eta = 1$	−	−	−
		$\eta > 1$	−	−	−
		$\eta < 1$	−	+/−	+/−
Hicks-Neutral	X	$\eta = 1$	−	0	−
		$\eta > 1$	−	−	−
		$\eta < 1$	−	+	+/−
Labor-Using	X	$\eta = 1$	−	+	+/−
		$\eta > 1$	−	+/−	+/−
		$\eta < 1$	−	+	+/−
Skill-Using	Y	$\eta = 1$	+	−	+/−
		$\eta > 1$	+	+/−	+/−
		$\eta < 1$	+	−	+/−
Hicks-Neutral	Y	$\eta = 1$	+	0	+
		$\eta > 1$	+	+	+
		$\eta < 1$	+	−	+/−
Labor-Using	Y	$\eta = 1$	+	+	+
		$\eta > 1$	+	+	+
		$\eta < 1$	+	+/−	+/−

[a] Notes: '+', '0', '−', and '+/−' refer to positive, zero, negative, and indefinite effect of the corresponding technical progress on the relative wage of unskilled labor. X and Y refer to the skill-intensive sector and the labor-intensive sector, respectively. η is the elasticity of substitution in demand.

does not require a factor-abundance ranking of the two countries. There will be some restrictions on factor abundance to guarantee diversified production. We describe Home's equations below. The equations for Foreign can be obtained by adding an asterisk to the variables and parameters in Home's equations. The production function of good X takes the form

$$X = F(L_x/a, H_x/b), \tag{1}$$

where L_x and H_x denote unskilled and skilled labor employed in the X sector, respectively. Production function $F(.)$ satisfies the neoclassical properties of constant returns to scale, positive and diminishing marginal products with respect to each input, and Inada conditions. The function contains two factor-augmenting productivity parameters, a and b. Production function (1) implies that the unit cost function of good X takes the form

$$c^x = c^x(aw_L, bw_H), \tag{2}$$

where w_L and w_H denote wage rates of unskilled and skilled labor, respectively.

The function $c^x(.)$ is linearly homogeneous and increasing at a diminishing rate in each wage rate. Similarly we specify the unit cost function of good Y as

$$c^y = c^y(\alpha w_L, \beta w_H), \tag{3}$$

where α and β are factor-augmenting productivity parameters of the Y sector.

2.1. Zero-profit conditions

We choose good Y as the numéraire and set its price to be one. Denote p as the price of good X in Home. Perfect competition leads to the following zero-profit conditions:

$$c^x(aw_L, bw_H) = p, \tag{4}$$

$$c^y(\alpha w_L, \beta w_H) = 1. \tag{5}$$

Define $\omega \equiv w_L/w_H$ as the relative wage of unskilled labor. Since the unit cost function is linearly homogeneous, we have $c^x(aw_L, bw_H) = w_H c^x(a\omega, b)$ and $c^y(\alpha w_L, \beta w_H) = w_H c^y(\alpha\omega, \beta)$. Dividing (4) by (5) yields:

$$\frac{c^x(a\omega, b)}{c^y(\alpha\omega, \beta)} = p. \tag{6}$$

Similarly we obtain from Foreign's zero-profit conditions:

$$\frac{c^x(a^*\omega^*, b^*)}{c^y(\alpha^*\omega^*, \beta^*)} = p^*. \tag{7}$$

We assume that the two countries engage in free trade and production is diversified in equilibrium (i.e., each country produces both goods). Diversified production requires that productivity-adjusted factor abundance is sufficiently similar between the two countries, which we assume to be true. The assumption of free trade is not essential; introducing tariffs in the model does not affect our analysis of the effects of technical progress on relative factor prices.

In a free-trade diversified equilibrium, $p = p^*$. Therefore, Eqs. (6) and (7) imply

$$\frac{c^x(a\omega, b)}{c^y(\alpha\omega, \beta)} = \frac{c^x(a^*\omega^*, b^*)}{c^y(\alpha^*\omega^*, \beta^*)}. \tag{8}$$

Eq. (8) defines a relationship between ω and ω^*. In Fig. 1, we depict this relationship as the ZZ curve.

The ZZ curve has a positive slope. The positive relationship between ω and ω^* is an implication of the Stolper-Samuelson theorem. Since good X is less intensive in unskilled labor than good Y, the Stolper-Samuelson theorem implies that both ω

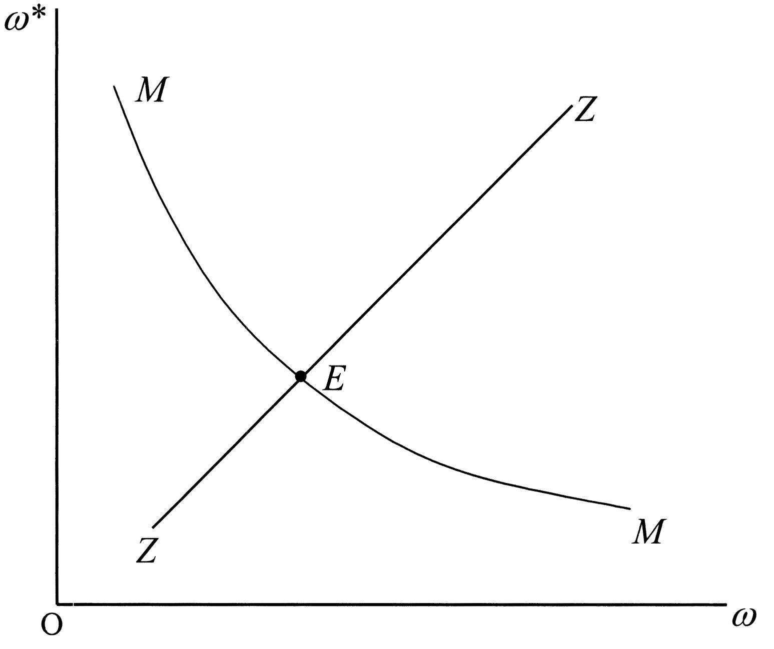

Fig. 1. Determination of relative factor prices.

and ω^* decrease as p increases; hence they are positively associated with each other.

The shape of the ZZ curve depends on how Home and Foreign differ in technologies. If Home and Foreign have identical technologies, or more generally, if Home and Foreign productivity differences are uniform across both factors and sectors, then the ZZ curve is a 45° line from the origin.[3] If Home and Foreign's productivity differences vary across factors but are identical for each factor across sectors, then the ZZ curve is a straight line from the origin whose slope depends on the productivity differences in skilled labor relative to that in unskilled labor.[4] For example, if Home's productivity advantage over Foreign is larger in skilled labor than in unskilled labor, then the ZZ curve is flatter than the 45° line. The ZZ curve is nonlinear if Home and Foreign's productivity differences vary across both factors and sectors. Since the shape of the ZZ curve is not essential for our analysis, we draw it as a straight line in Fig. 1.

[3]This is the case in which $t = \delta t^*$ for $t = a$, b, α, and β, where δ is a parameter measuring the uniform productivity difference. In this case, $\omega = \omega^*$.

[4]This is the case in which $a = \delta_L a^*$, $b = \delta_H b^*$, $\alpha = \delta_L \alpha^*$, and $\beta = \delta_H \beta^*$, where δ_L and δ_H are parameters measuring productivity differences in unskilled and skilled labor, respectively. In this case, $\omega = (\delta_H / \delta_L)\omega^*$.

2.2. Market clearing conditions

In this subsection we derive a second relationship between ω and ω^* from labor-market and commodity-market clearing conditions.

Partially differentiating the unit cost function with respect to w_H and w_L yields the demand for skilled and unskilled labor per unit of output, respectively. To simplify the notation, we define $l^x \equiv \partial c^x(aw_L, bw_H)/\partial(aw_L)$ and $h^x \equiv \partial c^x(aw_L, bw_H)/\partial(bw_H)$. Since $c^x(.)$ is homogeneous of degree one in aw_L and bw_H, both l^x and h^x are homogeneous of degree zero; therefore, we can write $l^x = l^x(a\omega, b)$ and $h^x = h^x(a\omega, b)$, and similarly $l^y = l^y(\alpha\omega, \beta)$ and $h^y = h^y(\alpha\omega, \beta)$. Assuming full employment, we specify Home's labor-market clearing conditions as:

$$al^x(a\omega, b)X + \alpha l^y(\alpha\omega, \beta)Y = L, \tag{9}$$

$$bh^x(a\omega, b)X + \beta h^y(\alpha\omega, \beta)Y = H, \tag{10}$$

where X and Y denote output of good X and good Y, respectively. Using Cramer's rule, we solve Y from (9) and (10). The solution is given by

$$Y = \frac{al^x(a\omega, b)H - bh^x(a\omega, b)L}{a\beta l^x(a\omega, b)h^y(\alpha\omega, \beta) - \alpha bh^x(a\omega, b)l^y(\alpha\omega, \beta)}. \tag{11}$$

We can show that $Y'(\omega) > 0$. Under the assumed production technologies, Y is negatively linked to p; under the assumed factor intensity ranking of the two goods, p is negatively linked to ω; hence Y is positively linked to ω. The sign of $Y''(\omega)$ depends on the form of the production function. For example, if the production function is Cobb-Douglas, then $Y''(\omega) < 0$.[5]

Turning to the demand side, we assume identical and homothetic preferences. For expositional convenience we specify Cobb-Douglas preferences in this section and consider non-Cobb-Douglas preferences in the next section. With Cobb-Douglas preferences, consumers spend a fixed share of income on each good. Let λ be the share of expenditure on good Y. We have:

$$D_y = \lambda w_H(\omega L + H), \tag{12}$$

where D_y denotes consumption of good Y. Using Eq. (5) and the property that $c^y(\alpha w_L, \beta w_H)$ is linearly homogeneous, we obtain:

$$w_H = \frac{1}{c^y(\alpha\omega, \beta)}. \tag{13}$$

Substituting (13) into (12), we have:

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[5] With Cobb-Douglas production technology, $Y'(\omega) = (-A\omega + B)/\omega^{1+\mu}$, where A and B are positive constants, and μ is the factor share of unskilled labor. It follows that $Y''(\omega) < 0$.

$$D_y = \frac{\lambda(\omega L + H)}{c^y(\alpha\omega, \beta)}. \tag{14}$$

We can show that $D_y'(\omega) < 0$.[6] An increase in ω raises the relative price of good Y, causing consumers to substitute consumption of good X for good Y. This substitution effect dominates the income effect under Cobb-Douglas preferences; hence D_y falls as ω rises. We can also show that $D_y''(\omega) > 0$ under Cobb-Douglas preferences.[7]

To describe commodity market equilibrium, we define Home's excess demand function of good Y as $M \equiv D_y - Y$. Using the expression for Y in Eq. (11) and the expression for D_y in Eq. (14), we can write:

$$M = M(\omega, a, b, \alpha, \beta). \tag{15}$$

Similarly, Foreign's excess demand for good Y is given by

$$M^* = M^*(\omega^*, a^*, b^*, \alpha^*, \beta^*). \tag{16}$$

World market for good Y clears when the world's excess demand for good Y equals zero:[8]

$$M(\omega, a, b, \alpha, \beta) + M^*(\omega^*, a^*, b^*, \alpha^*, \beta^*) = 0. \tag{17}$$

Eq. (17) defines a relationship between ω and ω^*. In Fig. 1, we depict this relationship as the MM curve. A point along the MM curve is a pair of relative factor prices that equalize import demand and export supply of each good.

The MM curve has a negative slope.[9] With a higher ω at home, there will be less unskilled labor used in both goods, which would lead to more production of the labor-intensive good Y so as to satisfy the full-employment conditions. In the meantime, a higher ω implies a lower demand for good Y. Therefore, a higher ω results in an excess supply of good Y from Home. For world commodity markets to clear, Foreign will produce less good Y, which is consistent with full employment in Foreign only if there is relatively more unskilled labor used in both goods, so that ω^* must be lower.

The shape of the MM curve depends on the forms of production functions and utility functions. In the case of Cobb-Douglas preferences and technologies, the

[6] $D_y'(\omega) = \lambda L \alpha l^y (h_Y - h)/[c^y(\alpha\omega, \beta)]^2 < 0$, where $h \equiv H/L$, and h_Y is the skill intensity of sector Y.

[7] This can be shown by differentiating $D_y'(\omega) = \lambda L \alpha l^y (h_Y - h)/[c^y(\alpha\omega, \beta)]^2$ and noting that $\partial l^y/\partial\omega < 0$, $\partial h_Y/\partial\omega > 0$, and $h_Y < h$.

[8] The condition for good X is redundant due to Walras' law.

[9] Totally differentiating (17) yields $d\omega^*/d\omega = -M'(\omega)/M^{*\prime}(\omega^*)$. Since $Y'(\omega) > 0$ and $D_y'(\omega) < 0$ (under Cobb-Douglas preferences), we have $M'(\omega) < 0$; similarly $M^{*\prime}(\omega^*) < 0$. Therefore, $d\omega^*/d\omega < 0$. A sufficient condition for $d\omega^*/d\omega < 0$ under non-Cobb-Douglas preferences is that the substitution effect in consumption dominates the income effect of a change in ω.

MM curve is convex.[10] Since the shape of the *MM* curve is not essential for our analysis, we draw it as a convex curve in Fig. 1.

2.3. Determination of relative factor prices

The intersection of the *MM* and *ZZ* curves determines relative factor prices in the two countries, ω and ω^*. We call the *MM-ZZ* diagram the factor-price determination diagram. The solutions of ω and ω^* can be written as:

$$\omega = \omega(a, b, \alpha, \beta, a^*, b^*, \alpha^*, \beta^*), \tag{18}$$

$$\omega^* = \omega^*(a, b, \alpha, \beta, a^*, b^*, \alpha^*, \beta^*). \tag{19}$$

We will determine the partial derivatives of these two functions in the following section.

3. Technical progress and relative factor prices

In this section we first define factor bias and sector bias of technical progress. We then investigate three cases: (1) local technical progress in a small open economy, (2) identical technical progress in an integrated world economy, and (3) non-identical technical progress in a world of two large open economies. This three-case procedure decomposes the total effect of technical progress into several components, which helps to explain the roles of factor bias and sector bias of technical progress in determining relative factor prices.

3.1. Factor bias and sector bias of technical progress

Sector bias refers to whether technical progress occurs in the skilled or unskilled labor-intensive sector. Factor bias refers to whether technical progress is biased in certain way toward skilled or unskilled labor.

The nature of factor bias determines how technical progress is classified. One classification is based on a factor-augmenting bias. Technical progress is labor-augmenting (skill-augmenting) if it raises output of a sector in the same way as an increase in the input of unskilled labor (skilled labor) in that sector. This classification is explicit in our model:

1. Labor-augmenting technical progress in sector X: a decrease in a;
2. Skill-augmenting technical progress in sector X: a decrease in b;
3. Labor-augmenting technical progress in sector Y: a decrease in α;

[10] This follows from $Y''(\omega) < 0$ and $D_y''(\omega) > 0$ shown in previous footnotes.

4. Skill-augmenting technical progress in sector Y: a decrease in β.

Technical progress can also be classified, as Hicks (1932) suggested, according to a factor-using bias. Technical progress is skill-using (labor-using) if it raises the intensity of skilled labor (unskilled labor) in a sector at constant relative factor prices.[11] In our model, skill intensities of sectors X and Y are given by

$$h_X = \frac{bh^x(a\omega, b)}{al^x(a\omega, b)}, \tag{20}$$

$$h_Y = \frac{\beta h^y(\alpha\omega, \beta)}{\alpha l^y(\alpha\omega, \beta)}. \tag{21}$$

Partially differentiating (20) and (21), we establish:

Lemma 1. *At constant relative factor prices,*

(*i*) $\partial h_X/\partial a < 0$ *if* $\sigma_X < 1$, $\partial h_X/\partial a = 0$ *if* $\sigma_X = 1$, *and* $\partial h_X/\partial a > 0$ *if* $\sigma_X > 1$;
(*ii*) $\partial h_X/\partial b < 0$ *if* $\sigma_X > 1$, $\partial h_X/\partial b = 0$ *if* $\sigma_X = 1$, *and* $\partial h_X/\partial b > 0$ *if* $\sigma_X < 1$;
(*iii*) $\partial h_Y/\partial \alpha < 0$ *if* $\sigma_Y < 1$, $\partial h_Y/\partial \alpha = 0$ *if* $\sigma_Y = 1$, *and* $\partial h_Y/\partial \alpha > 0$ *if* $\sigma_Y > 1$;
(*iv*) $\partial h_Y/\partial \beta < 0$ *if* $\sigma_Y > 1$, $\partial h_Y/\partial \beta = 0$ *if* $\sigma_Y = 1$, *and* $\partial h_Y/\partial \beta > 0$ *if* $\sigma_Y < 1$;

where σ_X *and* σ_Y *are the elasticities of factor substitution in sectors* X *and* Y, *respectively.*

Lemma 1 relates factor-using bias to factor-augmenting parameters. The relations depend on the elasticities of factor substitution. We have the following correspondences:[12]

1. Hicks-neutral technical progress in sector X: a decrease in a or b under $\sigma_X = 1$;
2. Skill-using technical progress in sector X: a decrease in a under $\sigma_X < 1$, or a decrease in b under $\sigma_X > 1$;
3. Labor-using technical progress in sector X: a decrease in a under $\sigma_X > 1$, or a decrease in b under $\sigma_X < 1$;
4. Hicks-neutral technical progress in sector Y: a decrease in α or β under $\sigma_Y = 1$;
5. Skill-using technical progress in sector Y: a decrease in α under $\sigma_Y < 1$, or a decrease in β under $\sigma_Y > 1$;

[11] Skill-using (labor-using) technical progress is also called labor-saving (skill-saving) technical progress.

[12] These correspondences are established under the assumption that only one of the factor-augmenting parameters changes. If both a and b can change, then technical progress is Hicks-neutral if a and b fall at the same rate under $\sigma_X \neq 0$; it is skill-using if a falls faster than b under $\sigma_X < 1$; and so on. This generalization would not affect our results.

6. Labor-using technical progress in sector Y: a decrease in α under $\sigma_Y > 1$, or a decrease in β under $\sigma_Y < 1$.

Both classifications of factor bias have been used in recent empirical studies of technical progress and relative wages. For example, Kahn and Lim (1998) estimated skill-augmenting technical progress in US manufacturing, while Berman et al. (1998) provided international evidence on the implications of skill-using technical progress. For reasons explained in Section 3.3, theoretical studies have favored Hicks' (1932) definition.

In what follows, we will present results for both classifications of technical progress, using factor-augmenting parameters. For each parameter, the reader may want to first link it to a type of factor-augmenting technical progress, and then link it to a type of factor-using technical progress with the help of the correspondences established above.

3.2. Local technical progress in a small open economy

We start with a case of local technical progress in a small open economy, which we call Home. In this case, world prices are determined in Foreign as if it were a closed economy. The relative commodity price p^* in Foreign corresponds to relative factor price ω^*. Since Home is too small to affect p^*, it takes ω^* as given. Thus, the *MM* curve is a horizontal line (Fig. 2).

Home's relative wage of unskilled labor, ω, is determined by

$$\frac{c^x(a\omega, b)}{c^y(\alpha\omega, \beta)} = p^*. \tag{22}$$

Totally differentiating (22), we establish:

Proposition 1. *In a small open economy, if local technical progress occurs in the skill-intensive sector, then the relative wage of unskilled labor falls* ($\mathrm{d}\omega/\mathrm{d}a > 0$, $\mathrm{d}\omega/\mathrm{d}b > 0$); *if it occurs in the labor-intensive sector, then the relative wage of unskilled labor rises* ($\mathrm{d}\omega/\mathrm{d}\alpha < 0$, $\mathrm{d}\omega/\mathrm{d}\beta < 0$).

Proposition 1 is a restatement of a well-known result in the international trade literature. It shows that sector bias solely determines the impact of local technical progress on relative factor prices in a small open economy. For example, technical progress in the skill-intensive sector, whether it is labor-augmenting (i.e. a decrease in a) or skill-augmenting (i.e. a decrease in b), will cause ω to fall; technical progress in the labor-intensive sector (i.e. a decrease in α or β), however, will cause ω to rise. Leamer (1998) emphasized this result.

Notice that the results of Proposition 1 are independent of the elasticities of factor substitution. Therefore, they apply equally to factor-using technical pro-

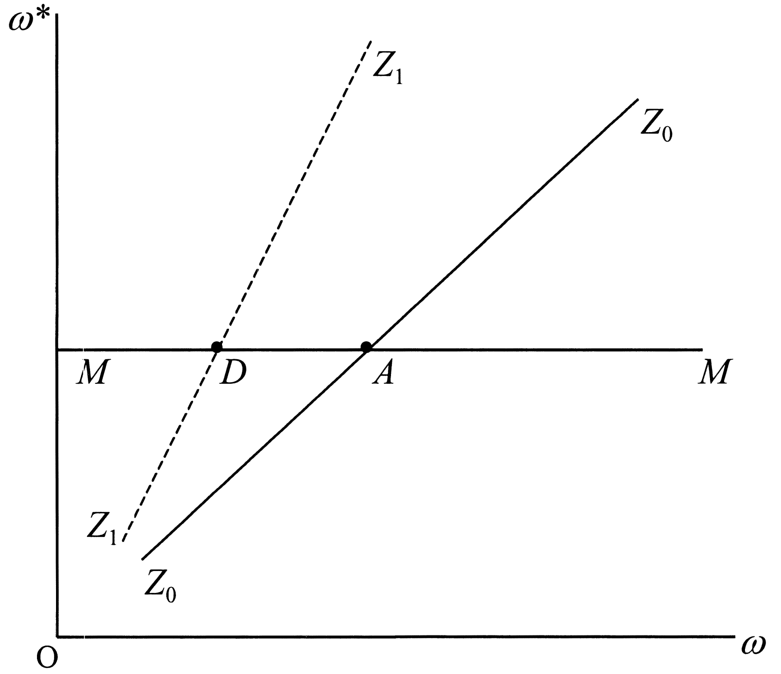

Fig. 2. Small open economy.

gress. For future reference, we reproduce the results of Proposition 1 in column (4) of Tables 1 and 2.

Fig. 2 illustrates the case of technical progress in the X sector. This technical progress shifts up Z_0Z_0 to Z_1Z_1, causing ω to decrease from A to D.

3.3. Identical technical progress in an integrated world economy

Next consider an integrated world economy. By definition, countries in an integrated world economy have identical technologies ($a = a^*$, $b = b^*$, $\alpha = \alpha^*$, and $\beta = \beta^*$) and hence factor prices are equalized under free trade ($\omega = \omega^*$). Let technical progress occur simultaneously in all countries at the same rate, which we call identical technical progress. With identical technical progress, $\omega = \omega^*$ always holds; hence the ZZ curve is always a 45° line starting from the origin (Fig. 3).

Identical technical progress in an integrated world economy is equivalent to technical progress in a closed economy. Jones (1965) investigated the effects of factor-using technical progress in a two-by-two general equilibrium model of a closed economy. A special case of the model was used by Krugman (2000) to emphasize the role of factor bias of technical progress in determining relative factor prices (discussed below).

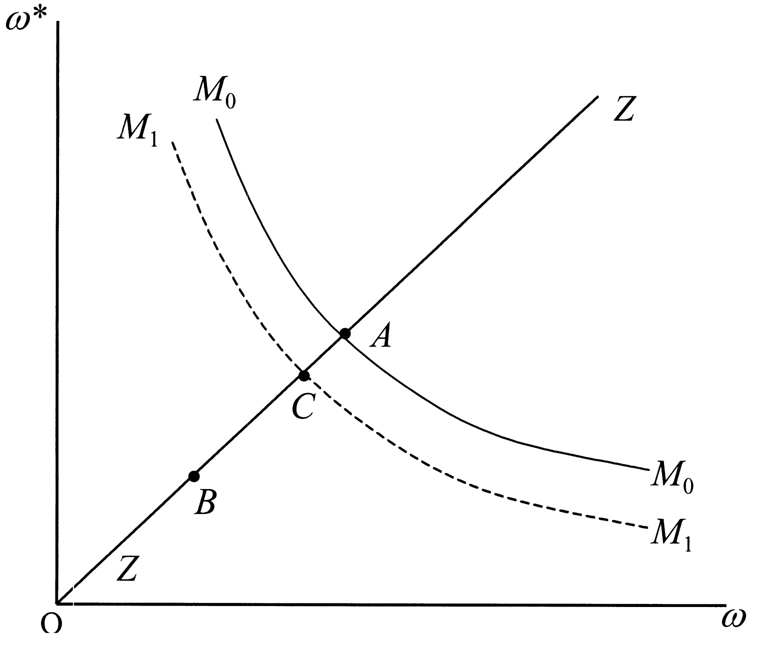

Fig. 3. Intergrated world economy.

Treating the world as one closed economy, we can solve the relations between relative factor prices and factor-augmenting parameters from

$$M(\omega, a, b, \alpha, \beta) = 0. \tag{23}$$

Totally differentiating (23), we establish:[13]

Proposition 2. *In an integrated world economy with Cobb-Douglas preferences,*

(i) $\mathrm{d}\omega/\mathrm{d}a > 0$ *if* $\sigma_X < 1$, $\mathrm{d}\omega/\mathrm{d}a = 0$ *if* $\sigma_X = 1$, *and* $\mathrm{d}\omega/\mathrm{d}a < 0$ *if* $\sigma_X > 1$;
(ii) $\mathrm{d}\omega/\mathrm{d}b < 0$ *if* $\sigma_X < 1$, $\mathrm{d}\omega/\mathrm{d}b = 0$ *if* $\sigma_X = 1$, *and* $\mathrm{d}\omega/\mathrm{d}b > 0$ *if* $\sigma_X > 1$;
(iii) $\mathrm{d}\omega/\mathrm{d}\alpha > 0$ *if* $\sigma_Y < 1$, $\mathrm{d}\omega/\mathrm{d}\alpha = 0$ *if* $\sigma_Y = 1$, *and* $\mathrm{d}\omega/\mathrm{d}\alpha < 0$ *if* $\sigma_Y > 1$;
(iv) $\mathrm{d}\omega/\mathrm{d}\beta < 0$ *if* $\sigma_Y < 1$, $\mathrm{d}\omega/\mathrm{d}\beta = 0$ *if* $\sigma_Y = 1$, *and* $\mathrm{d}\omega/\mathrm{d}\beta > 0$ *if* $\sigma_Y > 1$.

We can generalize the model to non-Cobb-Douglas homothetic preferences. When preferences are non-Cobb-Douglas, expenditure share λ changes with p. It can be shown that $\lambda'(p) > 0$ if $\eta > 1$, and $\lambda'(p) < 0$ if $\eta < 1$, where η is the elasticity of substitution in demand. Using Eq. (6) we obtain $p = p(\omega, a, b, \alpha, \beta)$. Substituting it into $\lambda = \lambda(p)$, we have $\lambda = \lambda(\omega, a, b, \alpha, \beta)$. Since consumption of

[13]Details of the proofs of Propositions 2 and 3 are available upon request.

good Y is a function of λ, excess demand for good Y is also a function of λ. Thus, Eq. (23) becomes:

$$M(\omega, a, b, \alpha, \beta, \lambda(\omega, a, b, \alpha, \beta)) = 0. \tag{24}$$

Totally differentiating (24), we establish:

Proposition 3. *In an integrated world economy with elastic substitution in demand* $(\eta > 1)$,

(i) $\mathrm{d}\omega/\mathrm{d}a > 0$ *if* $\sigma_X \leq 1$, *and the sign of* $\mathrm{d}\omega/\mathrm{d}a$ *is indeterminate if* $\sigma_X > 1$;
(ii) $\mathrm{d}\omega/\mathrm{d}b > 0$ *if* $\sigma_X \geq 1$, *and the sign of* $\mathrm{d}\omega/\mathrm{d}b$ *is indeterminate if* $\sigma_X < 1$;
(iii) $\mathrm{d}\omega/\mathrm{d}\alpha < 0$ *if* $\sigma_Y \geq 1$, *and the sign of* $\mathrm{d}\omega/\mathrm{d}\alpha$ *is indeterminate if* $\sigma_Y < 1$;
(iv) $\mathrm{d}\omega/\mathrm{d}\beta < 0$ *if* $\sigma_Y \leq 1$, *and the sign of* $\mathrm{d}\omega/\mathrm{d}\beta$ *is indeterminate if* $\sigma_Y > 1$.

The converse is true when substitution in demand is inelastic $(0 \leq \eta < 1)$.

Propositions 2 and 3 contain a set of relations between factor-augmenting parameters and relative factor prices in an integrated world economy. We find that the relations depend on both the elasticities of factor substitution and the elasticity of substitution in demand. For future reference, we reproduce these relations in columns (5)–(7) of Table 1.

Applying the correspondences between factor-using technical progress and factor-augmenting parameters (Section 3.1), we can express the relations in Propositions 2 and 3 in terms of factor-using technical progress. This establishes:

Proposition 4. *In an integrated world economy,*

(i) *if* $\eta = 1$, *skill-using technical progress lowers* ω, *labor-using technical progress raises* ω, *and Hicks-neutral technical progress has no effect on* ω;
(ii) *if* $\eta > 1$,*skill-using or Hicks-neutral technical progress in the X sector lowers* ω, *labor-using or Hicks-neutral technical progress in the Y sector raises* ω, *and other types of technical progress have ambiguous effect on* ω;
(iii) *if* $0 \leq \eta < 1$, *labor-using or Hicks-neutral technical progress in the X sector raises* ω, *skill-using or Hicks-neutral technical progress in the Y sector lowers* ω, *and other types of technical progress have ambiguous effect on* ω.

Proposition 4 shows that the effects of factor-using technical progress are independent of the elasticities of factor substitution.[14] This feature makes the factor-using classification appealing for theoretical analyses of the subject. One

[14]The results stated in Proposition 4 were established in Amano (1964) and Jones (1965) who defined factor-using technical progress directly without relating it to factor-augmenting parameters.

needs to understand, however, that the simplicity comes from the particular way factor-using technical progress is defined. In contrast to factor-augmenting technical progress which is defined *before* factor intensities respond to technical progress, factor-using technical progress is defined *after* factor intensities respond to technical progress at initial relative factor prices; this partial adjustment of factor intensities preempts the effects of the elasticities of factor substitution.[15]

Proposition 4(i) shows that under Cobb-Douglas preferences, factor-using bias solely determines the impact of technical progress on relative factor prices in a closed economy. Krugman (2000) emphasized this result in his debate with Leamer (1998). To better understand this and other results, we decompose the total effect of technical progress into two components: (1) a direct effect defined as the effect of technical progress on relative factor prices holding relative commodity prices constant, and (2) an indirect effect defined as the effect of technical progress on relative factor prices that works through relative commodity prices.

The direct effect is precisely the effect of local technical progress in a small open economy. Therefore, the direct effect depends solely on sector bias. The indirect effect is an additional effect that occurs in a large economy where commodity prices respond to technical progress. The indirect effect works through the following mechanism: Technical progress affects relative commodity supplies, which affect relative commodity prices, which in turn affect relative factor prices through the Stolper-Samuelson mechanism. Findlay and Grubert (1959) examined the effects of factor-using progress on relative commodity prices in the two-by-two general equilibrium model. Their results show that the indirect effect depends on both sector bias and factor-using bias.

Adding the indirect effect to the direct effect, we obtain the total effect of technical progress on relative factor prices in a closed economy. A boundary case is Cobb-Douglas preferences in which the indirect effect offsets the direct effect such that only factor-using bias matters for relative wages (Krugman, 2000). Proposition 4(ii)(iii) shows, however, that sector bias also affects relative wages when preferences are non-Cobb-Douglas. The results of Proposition 4 are reproduced in column (5) of Table 2.

In Fig. 3 we illustrate a case of identical technical progress in a two-country integrated world economy. The *ZZ* curve is independent of any technical progress, while the *MM* curve shifts in response to technical progress. Holding world commodity prices fixed, technical progress in sector X causes both ω and ω^* to decrease, implying a 'global direct effect' from A to B. Since the technical progress causes world commodity prices to adjust, there is also a 'global indirect effect.' If the technical progress is Hicks-neutral and preferences are Cobb-Douglas, then the global indirect effect is from B to A, which exactly offsets the global direct effect AB, leaving ω and ω^* unaffected. If the technical progress is

[15] See Jones (2000) for an elaboration of this point.

skill-using and $\eta \geq 1$, then the global indirect effect is from *B* to *C*, which partially offsets the global direct effect *AB*, implying lower ω and ω^* in the new equilibrium.[16]

3.4. Non-identical technical progress in a world economy

In this subsection we consider global technical progress that occurs at different rates in the two countries, which we call non-identical technical progress.[17] For example, suppose both a and a^* decrease such that $(-\hat{a}) > (-\hat{a}^*)$, where a hat over a variable means a proportional rate of change. Analytically, we can decompose any non-identical technical progress into two parts: identical technical progress in both countries and local technical progress in the country where technical progress is faster. In the above example, the non-identical technical progress can be decomposed into identical technical progress at rate $(-\hat{a}^*)$ in both countries and local technical progress at rate $(\hat{a}^* - \hat{a})$ in Home. To derive the effects of non-identical technical progress, we first examine local technical progress in a large open economy (Home by assumption). The relations between technical progress and relative factor prices in this economy can be solved from

$$\frac{c^x(a\omega, b)}{c^y(\alpha\omega, \beta)} = p(\omega^*), \tag{25}$$

$$M(\omega, a, b, \alpha, \beta) + M^*(\omega^*) = 0, \tag{26}$$

where $p'(\omega^*) < 0$. In Fig. 4, Eq. (25) defines the *ZZ* curve, and Eq. (26) defines the *MM* curve.

We observe that at any given ω^*, the relations between ω and the productivity parameters in Eq. (25) are the same as those in the case of local technical progress in a small open economy. Therefore, Proposition 1 can be used to determine how the *ZZ* curve shifts horizontally. For example, when technical progress occurs in the *X* sector of Home, the *ZZ* curve shifts to the left from Z_0Z_0 to Z_2Z_2 (Fig. 4).

Similarly, we find that at any given ω^*, the relations between ω and the productivity parameters in Eq. (26) are the same as those in the case of identical technical progress in an integrated world economy. Therefore, Propositions 2, 3 and 4 can be used to determine how the *MM* curve shifts horizontally. When

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[16] It is possible for point *C* to locate below point *B*. The reason is as follows: In the case of skill-using technical progress in the *X* sector, the skill intensity of good *X* rises *at constant relative factor prices*. However, the skill intensity of good *X* may rise or fall *in equilibrium* because of factor substitution in response to changes in relative factor prices. As a result, the relative supply of good *X* may rise or fall, and therefore the indirect effect may be positive or negative. See Findlay and Grubert (1959) for details.

[17] Throughout the paper we assume diversified production. This requires the two countries to have sufficiently similar effective factor abundance (i.e. factor abundance expressed in productivity equivalent units) in the presence of non-identical technical progress.

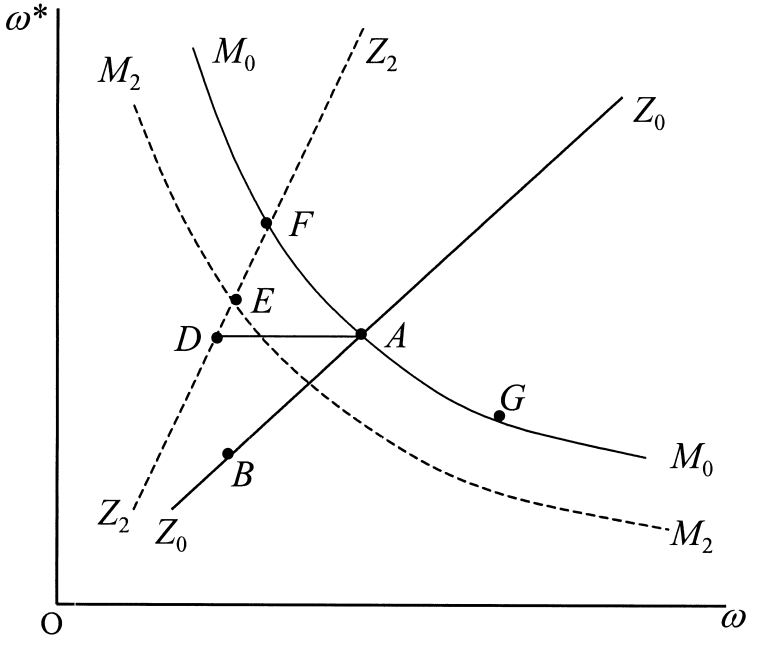

Fig. 4. Large open economies.

skill-using technical progress occurs in the X sector of Home, for example, the MM curve in Fig. 4 shifts to the left from M_0M_0 to M_2M_2 (assuming that $\eta \geq 1$).

The above discussions suggest a convenient way to determine the sign of the effect of any local technical progress on ω in a large open economy. The sign can be obtained by adding the sign of the effect in the 'small open economy case' (i.e. shift of the ZZ curve) to that in the 'integrated world economy case' (i.e. shift of the MM curve). In Table 1, column (8) is obtained by adding column (4) to column (5), column (9) is obtained by adding column (4) to column (6), and column (10) is obtained by adding column (4) to column (7).[18] This establishes:

Proposition 5. *Consider local technical progress in a large open economy.*

(*i*) *If* $\eta \geq 1$, *labor-augmenting technical progress in sector X lowers* ω *under* $\sigma_X \leq 1$, *skill-augmenting technical progress in sector X lowers* ω *under* $\sigma_X \geq 1$, *labor-augmenting technical progress in sector Y raises* ω *under* $\sigma_Y \geq 1$,

[18]Table 1 does not show the effects of local technical progress in Home on relative factor prices in Foreign. The signs of these effects can be easily obtained by subtracting column (4) from each of columns (5)–(7).

skill-augmenting technical progress in sector Y raises ω under $\sigma_Y \leq 1$, and the effect on ω is ambiguous in other cases;
(*ii*) *If $\eta < 1$, factor-augmenting technical progress has an ambiguous effect on ω.*

Similarly we add column (4) to column (5) in Table 2 to obtain column (6), which shows the effects of factor-using technical progress in a large open economy. This establishes:

Proposition 6. *Consider local technical progress in a large open economy.*

(*i*) *If $\eta \geq 1$, skill-using or neutral technical progress in sector X lowers ω, labor-using or neutral technical progress in sector Y raises ω, and the effect on ω is ambiguous in other cases*;
(*ii*) *If $\eta < 1$, factor-using technical progress has ambiguous effect on ω.*[19]

In Fig. 4 we illustrate a case of local technical progress in Home. Initially the *ZZ* curve is Z_0Z_0 and the *MM* curve is M_0M_0. Suppose technical progress occurs in sector *X*; then the *ZZ* curve shifts to the left. Let the technical progress be skill-using and $\eta \geq 1$; then the *MM* curve also shifts to the left. The new equilibrium is at point *E*. We can decompose the total effect from *A* to *E* into an effect from *A* to *D* and an additional effect from *D* to *E*. It is useful to note that the effect *AD* is exactly the 'local direct effect' in the case of a small open economy (Fig. 2). Because Home is a large country, there is an additional *global* indirect effect *DE*.

It is now straightforward to obtain the effects of non-identical technical progress. Assuming $\omega = \omega^*$ in the initial world equilibrium,[20] any non-identical progress can be decomposed into identical progress in an integrated world economy and local progress in the country where technical progress is faster (Home by assumption). The sign of the effect of non-identical progress on Home's relative factor prices can be obtained by adding the sign of the effect of identical progress in an integrated world economy to the sign of the effect of local progress in a large open economy. It turns out that the results are exactly the same as those of local technical progress in a large open economy.[21] In Table 1, adding column (5) to column (8) yields a column identical to column (8), adding column (6) to

[19]Proposition 6 extends Davis (1998) who showed the case of local factor-using technical progress in a large open economy with Leontief production technology.

[20]If the initial world equilibrium is characterized by $\omega \neq \omega^*$, then we need to derive the effects of identical technical progress in a non-integrated world economy, which we do not attempt here.

[21]This is not surprising since identical technical progress just adds an additional shift of the *MM* curve in the same direction of the shift of the *MM* curve due to local technical progress in a large country.

column (9) yields a column identical to column (9), and adding column (7) to column (10) yields a column identical to column (10). In Table 2, adding column (5) to column (6) yields a column identical to column (6).

The concept of non-identical technical progress helps to clarify a condition for Krugman's (2000) special case in which *only* factor bias matters for relative factor prices. Consider technical progress that is global. Suppose preferences are Cobb-Douglas, technical progress is Hicks-neutral, and the initial world equilibrium is an integrated equilibrium. In Fig. 4, if technical progress is identical in the two countries, then it causes a direct effect from A to B and an indirect effect from B to A. These two effects offset each other such that only the factor bias matters for relative factor prices (Krugman, 2000). However, if technical progress is faster in Home, then it implies an extra amount technical progress that is local in Home. If this local technical progress occurs in sector X, then Z_0Z_0 shifts up to Z_2Z_2, implying an effect from A to F. If it occurs in sector Y, then the effect is from A to G. This shows that sector bias matters in the boundary case of Cobb-Douglas preferences.

3.5. Factor bias vs. sector bias

In this subsection we summarize the model's implications on the roles of factor bias and sector bias of technical progress in determining relative factor prices.[22] When does factor bias matter for relative factor prices? The model shows that the factor bias matters as long as technical progress has a non-zero indirect effect through commodity prices. Only in two extreme situations does factor bias play no role: (1) local technical progress in a small open economy, and (2) infinitely elastic substitution in demand in a world economy. In both these cases technical progress leaves relative commodity prices unchanged.

When does sector bias matter for relative factor prices? The model identifies two situations. First, sector bias matters when preferences are non-Cobb-Douglas. In an integrated world economy, technical progress has a global direct effect that depends on sector bias, and a global indirect effect that depends on both sector bias and factor bias. With Cobb-Douglas preferences, the two effects offset each other such that relative factor prices depend only on the factor bias (Krugman, 2000). With non-Cobb-Douglas preferences, the indirect effect may reinforce the direct effect, reverse it in part, or more than offset it. To see it intuitively, we note that relative commodity prices are determined at the intersection of a relative-supply curve and a relative-demand curve, and the slope of the relative-demand curve is inversely related to the elasticity of substitution in demand. Technical progress causes a shift of the relative-supply curve; hence its effect on relative

[22]Following the literature, we discuss sector bias vs. factor-*using* bias. The discussion can be extended to factor-augmenting bias which adds an additional dimension related to elasticities of factor substitution.

commodity prices is captured by a movement along the relative-demand curve. When substitution in demand is inelastic ($\eta < 1$), the relative-demand curve is steep, which implies a large indirect effect that dominates the direct effect. When substitution in demand is elastic ($\eta > 1$), the relative-demand curve is flat, which implies a small indirect effect that is dominated by the direct effect. Both cases deviate from the boundary case of Cobb-Douglas preferences, and hence the sector bias affects relative factor prices. The larger the deviation of η from one, the more important the sector bias. In the extreme case of infinite elasticity of substitution in demand, relative commodity prices will be independent of relative consumptions, and hence each country would behave as if it were a small open economy facing fixed relative commodity prices; in this case relative factor prices depend only on the sector bias.

Second, sector bias matters when technical progress is non-identical in a world economy. When countries experience different rates of technical progress, there will exist a local direct effect due to the extra amount of technical progress that occurs in the country where technical progress is faster. Thus, the case of Leamer (1998) is only a special example of the local direct effect. The presence of the local direct effect does not require that thecountry is small and that the technical progress is local; all it requires is that technical progress is non-identical across countries. Only in one extreme case does sector bias play no role: technical progress is global *and* identical across countries in an integrated world economy with Cobb-Douglas preferences.

4. Conclusion

In this paper we investigated the effects of technical progress on relative factor prices in the $2\times2\times2$ Heckscher-Ohlin model. Classifying technical progress according to factor-augmenting bias, factor-using bias, and sector bias, we derived a complete set of relations between technical progress parameters and relative factor prices. These results complement those in the existing literature and provide some guidance for empirical and computational investigations of the effects of technical progress on relative wages. They show that for technical progress to lower the relative wage of unskilled labor, certain restrictions on the values of the elasticities of substitution in demand and in production must be met. We addressed the issue of whether it is the factor bias or the sector bias of technical progress that matters for relative factor prices. In a world economy where technical progress is global and non-identical across countries, we found that both the factor bias and the sector bias play a role in the determination of relative factor prices. On the one hand, we showed that Krugman (2000) was right in emphasizing that technical progress is global and it causes adjustments in commodity prices that would affect relative factor prices in a way dependent on the factor bias. On the other hand, we showed that the local direct effect responsible for Leamer's (1998) result is not limited to a small open economy; when technical progress is global but non-identical, the local direct effect will be present and therefore the sector bias will play a role. These results reconcile the views in the literature and clarify the conditions for factor bias and sector bias to impact relative factor prices.

Acknowledgements

I would like to thank Robert Feenstra and two anonymous referees for very helpful comments and suggestions, Alan Deardorff, David Denslow, Elias Dinopoulos, Ronald Findlay, Ronald Jones, Steven Slutsky, Douglas Waldo, and participants at the Midwest International Trade Meetings (Ann Arbor, 1998) for useful comments on earlier versions of the paper, and the Warrington College of Business Administration of the University of Florida for financial support. I am responsible for all remaining errors.

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